Linear Stability Analysis of Harmonic Oscillator in The Domain of Super Conductivity

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Abstract:

Harmonic oscillator or tank circuit is an idealized model because there is no dissipation of energy due to resistance. Tank circuit is used for creating signals at a certain frequency from a compound signal. In RLC circuit the resistor in series will start throwing away the power that should go to the load and due to unwanted resonance it would affect the stability of the system and it creates audible noise in RF circuits. In order to encounter this situation an LC circuit is used and this circuit will not waste that power. The technique used for analyzing the behavior mathematically and graphically is linear stability analysis. Linear stability analysis tells us about the behavior of a system near an equilibrium point. Linear stability analysis of harmonic oscillator in the domain of superconductivity determines the stability of fixed points in a circuit to visualize the qualitative analysis of resonance frequency when an LC circuit is operated from an external supply.

Keywords: *LC Circuit, Resonance, Linear Stability Analysis, Transfer function*

I. Introduction

The LC circuit, oscillating at its natural resonant frequency can store electrical energy. LC circuits are used either for producing signals at a certain frequency or picking out a signal at a specific frequency from a additional composite signal. The purpose of an LC circuit is usually to oscillate with minimal damping. LC circuits are often used as filters [1], [2], [3], [4] and [5].

The $\frac{L}{C}$ ratio is one of the aspects that determine their quality factor. Resonance occurs when a LC circuit is driven from an external supply at angular frequency at which the inductive and capacitive reactances are equal in magnitude. Resonance is a phenomenon in which a vibrating system drives another system to oscillate with greater amplitude. In LC circuit the capacitor stores energy in its electrical field and inductor stores energy in its magnetic field. LC circuit is given in fig. 1.

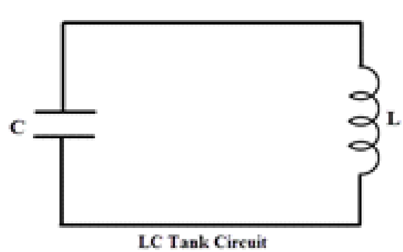

Fig. 1. LC Circuit

LC circuits perform as electronic resonators, which is very important component in many applications like filters, tuners and mixers [6]. The most extensively prominent use of tank circuits is tuning radio transmitters and receivers. For example, when we tune a radio to a particular station, the LC circuits are set at resonance for that particular carrier frequency. The characteristic frequency of an LC circuit is the frequency at which substantial amplitudes are developed when a driving force is connected at that frequency. In a LC circuit, electric charge oscillates forward and backward simply like the situation of a mass on a spring oscillates [7], [8] and [9]. The main advantage of this circuit is to make sure stability and in order to analyze the system near an equilibrium point; the linear stability analysis technique is used. This methodology is convenient to find out the behavior of dynamic system. RLC circuit can produce unwanted noise which can affect the whole dynamic system and in order to eradicate this noise tank circuit is used. The main concern of this research is to evaluate the behavior of tank circuit mathematically and graphically.

II. Related Work

Department of Physics and Astronomy, University of North Carolina at Greensboro investigate a simple variation of the series RLC circuit in which anti-parallel diodes replace the resistor. This results in a damped harmonic oscillator with a nonlinear damping term. A set of nonlinear differential equations for the oscillator circuit is derived and integrated numerically for comparison with circuit measurements. Unlike the standard RLC circuit, the behavior of this circuit is amplitude dependent [10]. Recently in Geneva researchers derive a relationship to find out the uniqueness of stable oscillations, for this purpose they used nonlinear analysis of second order circuit [11]. In Madrid Spain the researchers analyze the oscillator with linear analysis. Common collector method is used to illustrate this method and linear methods for oscillator requires some verification as well [12]. In linear active circuits considering switched resistor is the problem of stability analysis. The analysis of resistor switched circuits transfer function has poles in the left plane and the stability of the circuit is examined. The stability of the second order circuit was anticipated and verified for any suitably confined DC input. It is found that there exists a class of AC inputs for this circuit is unstable over utmost of the dynamic range of the input. The methods were offered for the second order system to higher order systems and the analysis of second order linear system with a piecewise linear feedback block showing chaotic oscillations. The requirement of a feedback block which cannot be a function is indicated. Stability analysis is one of the vital experiments in the design of large linear analog circuits with complex multi-loop arrangements.

III. Methods and Experimental Analysis

Time domain solution of LC circuit by Kirchhoffs voltage law is sum of capacitor and inductor voltage must be zero.

$$V_C + V_L = 0 \tag{1}$$

By Kirchhoff's current law we can say that

$$I_c = I_L \tag{2}$$

Frpom the constitutive relations for the circuit elements, we also know that

$$V_L\,(t) = L\frac{dI_L}{dt} \tag{3}$$

$$I_c\,(t) = C\,\frac{dv_C}{dt} \tag{4}$$

Rearraging and substituting gives the second order differential equation.

$$\frac{d^2}{dt^2}I(t) + \frac{1}{LC}I(t) = 0 \tag{5}$$

The resonant angular frequency is defined as

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{6}$$

By using this expression in above differential equation

$$\frac{d^2}{dt^2}I(t) + \omega_0^2 I(t) = 0 \tag{7}$$

Differential equations designate the development of systems in continuous time. In ordinary differential equation (ODE) there is only one independent variable [13] and [14]. Consider the previous ordinary differential equation in which current is a function of time.

$$\frac{d^2}{dt^2}I(t) + \omega_0^2 I(t) = 0 \tag{8}$$

Let x = I(t) and use this in above equation as $\dot{x} = x_1$ and further for second derivative $\ddot{x} = \dot{x}_1$
Final equation in terms of linear system is

$$\dot{x}_1 + \omega_0^2 x = 0 \tag{9}$$

Equivalent system is said to be3 linear.

$$\dot{x}_1 = -\omega_0^2 x \tag{10}$$

Because the "x" on the right hand side appear to the first power only. Otherwise system would be nonlinear.

Linear Stability Analysis is a method to determine the stability of fixed points. Consider the equation $0 = -\omega_0^2 x$ to find out the fixed point which is $x^* = 0$. We are considering the term ω_0^2 as constant to find out the derivative of previous equation which is

$$f'(x) = -\omega_0^2 \tag{11}$$

Inference of above equation is $f'(0) = -\omega_0^2$
and we know that $f'(x) < 0$ so the function is decreasing. An equilibrium solution is a constant solution of the system and is $-\omega_0^2 x = 0$ such a system has exactly one solution located aqt the origin. Graphs of $\dot{x}_1 = -\omega_0^2 x$ are situated below.

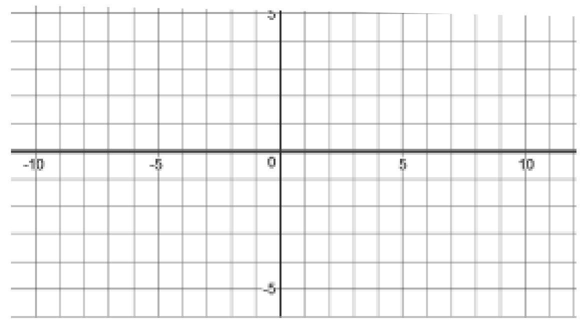

Fig.2. For $\omega_0 = 0$

Above is a straight line graph when parameter is zero which is resonance in electrical LC circuit. From above ω_0^2 is also zero this is only possible in LC$\rightarrow\infty$circuit $\frac{1}{LC}\rightarrow 0$when value of and for zero resonance value.

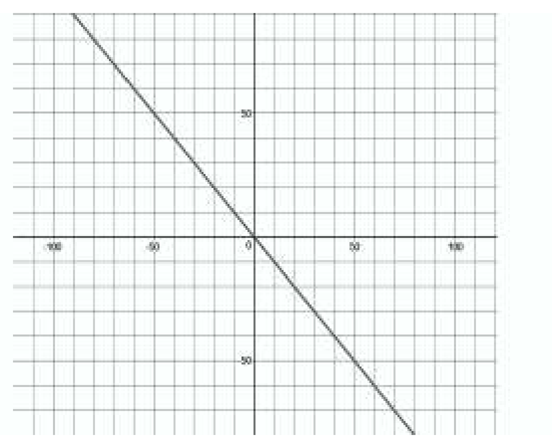

Fig.3 For $\omega_0 > 0$& $\omega_0 < 0$

If $\omega_0 > 0$ or it is equal to +1 then the graph looks like Fig.3 and the fixed point is at the origin which is stable. This graph is same for $\omega_0 = -1$ and the behavior of fixed point is also same in this case to obtain some pictorial visualization of mathematical equation.

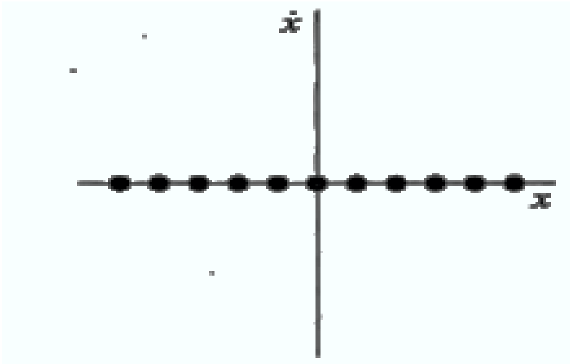

Fig. 4 when $\omega_0 = 0$

When value of parameter is zero there is a whole line of fixed points; perturbations neither grow nor decay.

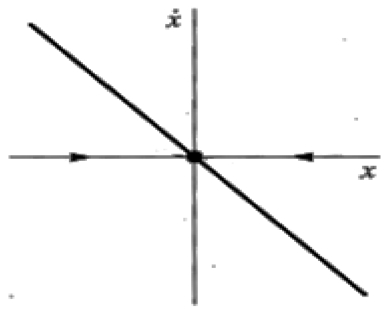

Fig.5 For $\omega_0 > 0$& $\omega_0 < 0$

Resonance parameter is greater or less than zero then the fixed point is stable or attracting fixed point.

IV. Transfer Function

Transfer function of LC circuit is complex voltage divider.

$$\text{Transfer Function} = \frac{\frac{1}{sC}}{sL + \frac{1}{sC}} = \frac{1}{s^2LC + 1} = \frac{1}{1 - \omega^2 LC} \qquad (12)$$

For large values of L, C and ω the transfer function becomes negative. In this case the signal is 180 degrees out of phase with the input. At $\omega = 0$ the value of transfer function is one.

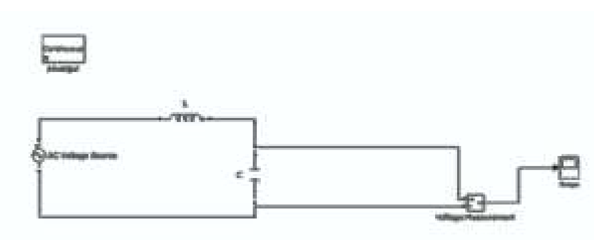

Fig.6. MATLAB Simulink Model

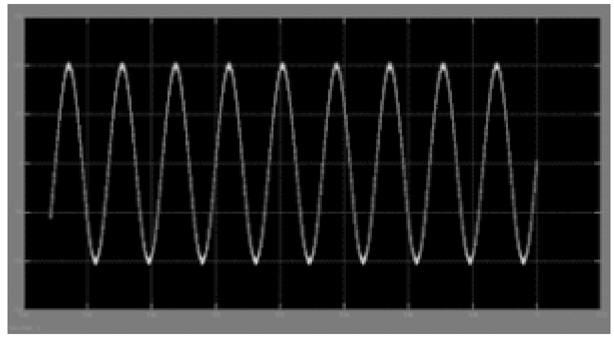

Fig. 7. Output Waveform

Simulation of LC circuit gives the realistic approach towards output voltage by setting the initial value zero of capacitor voltage and inductor current. Also input signal of 100 volts peak to peak to analyze the output waveform across the capacitor with 60 Hertz supply frequency. The circuit forms a harmonic oscillator. Introducing the resistance increases the decay of these oscillations. Output waveform corresponds to zero damping because amplitude doesn't change with time. Damping factor is also zero $\zeta = 0$ which indicates that the system is undamped.

Now $Q = \frac{X_L \text{ or } X_C}{R} = \infty$ because $R=0$ and quality factor is very large so the circuit has very low damping and it will vibrate longer. Higher Q indicates a lower rate of energy loss relative to stored energy[15], [16] and [17].

V. Step Response

If an ideal DC battery gets connected to ideal de-energized LC circuit, transient is an ideal sine current. Since there's no damping, it will oscillate forever [18], [19] and [20].

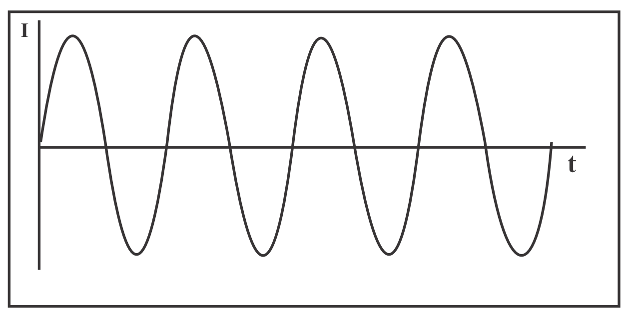

Fig. 8. LC Network Response

VI. Result Discussion

Due to absence of resistor the circuit is in the domain of superconductivity and there is no energy dissipation. When $\omega = 0$ and $\omega > 0$ or $\omega < 0$ then oscillations are stable and undamped. Quality factor is high in this circuit which indicates that the ratio of energy loss is very low. Simulation of basic tank circuit indicates that the oscillations are undamped.

VII. Conclusion

When resonance frequency is greater or less than zero then the fixed point is stable which means oscillations are stable and amplitude does not increase. At zero value of resonance frequency perturbations do not affect stability of oscillations. Output voltage waveform corresponds to zero damping because amplitude doesn't change with time. The polarity of the voltage varies as the energy is passed from side to side between the capacitor and inductor generating an AC type sinusoidal voltage.

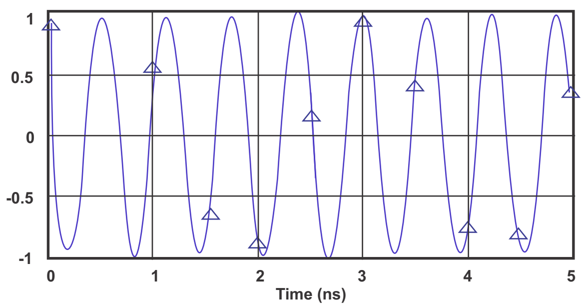

Fig. 9. Stable Oscillation

When we tune a radio to a specific station, the LC circuits are fixed at resonance for that specific carrier frequency. It is used for giving the essential positive feedback for supporting the oscillations.

VIII. Future Work

The voltage magnification that takes place at resonance is given the image Q and the "Q Factor" (the voltage magnification) of LC Band Pass and Band Stop filter circuits as an example, controls the "rejection", the ratio of the desired to the undesirable frequencies that may be accomplished through the circuit. The results of voltage magnification are particularly beneficial as they are able to offer magnification of AC sign voltages the use of simplest passive components, i.e. without the demand for any external power supply.

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